Branes Intersecting at Angles

Micha Berkooz, Michael R. Douglas and Robert G. Leigh

Department of Physics and Astronomy

Rutgers University, Piscataway, NJ 08855-0849

berkooz, mrd, leigh@physics.rutgers.edu

We show that configurations of multiple D-branes related by SU(N) rotations will preserve unbroken supersymmetry. This includes cases in which two D-branes are related by a rotation of arbitrarily small angle, and we discuss some of the physics of this. In particular, we discuss a way of obtaining 4D chiral fermions on the intersection of D-branes.

We also rephrase the condition for unbroken supersymmetry as the condition that a 'generalized holonomy group' associated with the brane configuration and manifold is reduced, and relate this condition (in Type IIA string theory) to a condition in eleven dimensions.

1. Introduction

Dirichlet (D) branes are BPS states and break half of the available supersymmetries [1,2]. By adding D-branes filling space-time to a string compactification, one thereby obtains models with additional gauge symmetry and less supersymmetry. Configurations containing multiple D-branes break supersymmetry further, and can leave interesting fractions such as 1/4 or 1/8. Such models have been studied in many recent works. [3,4,5,6,7,8]

So far, only D-branes intersecting at right angles have been studied. An indication that this is not the only possibility comes from considering the compactification on the torus $T^4 \times \mathbb{R}^6$ with a p-brane at a point in T^4 and a wrapped (p+4)-brane, a configuration which preserves 1/4 of the supersymmetry. One can T-dual any of the dimensions of the torus to produce intersecting D-branes wrapped around some of its dimensions, for example (p+2)-branes wrapped around two 2-planes. If the metric components $g_{ij} \neq 0$ for $i \neq j$, they will intersect at an angle. It is clear however that this is still a BPS state.

In this paper, we study more general configurations of D-branes. We find that the condition for intersecting D-branes to preserve supersymmetry is closely related to the well-known condition for a d-dimensional curved manifold to preserve supersymmetry: a generalized holonomy (which we will define) must be contained in a SU(d/2) subgroup of SO(d). This condition can be satisfied with arbitrary angles of intersection. We derive this condition and give examples of solutions in section 2.

Open strings joining two such D-branes are quasi-localized: as the opening angle θ decreases, their allowed spread from the intersection point increases. In section 3, we consider the world-sheet analysis of these strings, while in section 4 we discuss the possibility of describing these configurations by starting with the space-time effective Lagrangian for parallel D-branes and turning on a linearly increasing displacement. We show that these quasi-localized open string states form a Kaluza-Klein-like tower with spacing proportional to the angle. Section 3 also discusses 4D chiral fermions on the intersection of D-branes.

We then study T-duality, and find that the angles between branes are dual to orthogonal branes in certain spacetime backgrounds. In the final section, we give an 11-dimensional interpretation of our results.

2. Supersymmetry

Type II string theory has $\mathcal{N}=2$ supersymmetry in d=10 with parameters ε and $\tilde{\varepsilon}$, for left and right movers. In the presence of a D-brane, an unbroken supersymmetry must satisfy¹

$$\tilde{\varepsilon} = \prod_{i} e_{i}^{\mu} \Gamma_{\mu} \ \varepsilon \tag{2.1}$$

where e_i is an orthonormal frame spanning the D-brane. Let us refer to this product of Γ -matrices as Γ_D .

In the presence of several D-branes, an unbroken supersymmetry must satisfy the condition (2.1) for each brane. Two D-branes related by a rotation R therefore lead to the condition

$$\Gamma_D \varepsilon = R^{-1} \Gamma_D R \varepsilon \tag{2.2}$$

which will have a solution if

$$\det(\Gamma_D - R^{-1}\Gamma_D R) = 0. \tag{2.3}$$

Clearly the common dimensions of the branes do not affect the argument, so we will assume for definiteness that the time dimension is common to all the branes. Hence, in what follows, when we refer to the dimension of a brane, we will give only the (Euclidean) dimensions of interest.²

As is usual in supersymmetric compactification, the results will be most simply phrased in terms of complex structure of the target space. We work in N complex dimensional space with coordinates z^i . We define raising and lowering operators

$$a_k^{\dagger} = \frac{1}{2}(\Gamma_{2k-1} - i\Gamma_{2k})$$

 $a^k = \frac{1}{2}(\Gamma_{2k-1} + i\Gamma_{2k})$ (2.4)

The metric is (-,+,+,...). $\Gamma_0 = \begin{pmatrix} 0 & -1 \\ +1 & 0 \end{pmatrix}$, $\Gamma_i = \begin{pmatrix} 0 & \gamma_i \\ \gamma_i & 0 \end{pmatrix}$, where γ_i are real symmetric matrices.

² Thus a 2-brane is really a (p+2)-brane, having p+1 additional dimensions. We trust that this will confuse the reader.

acting on the 2^N -dimensional sum of the two spinor representations. Define the 'vacuum' $|0\rangle$, satisfying $a^k|0\rangle = 0$. An SU(N) rotation $z^i \to R^i_j z^j$ acts on the Clifford algebra as

$$a^{k} \to R^{k}{}_{l} a^{l}$$

$$a^{\dagger}_{k} \to R^{\dagger}{}_{k}{}^{l} a^{\dagger}_{l}.$$

$$(2.5)$$

If the branes are embedded in (say) a Calabi-Yau, this division into creation and annihilation operators is determined by the manifold. If not, we can choose complex coordinates in whatever fashion we like. We will find below that two branes preserve a common supersymmetry when they are related by an SU(N) rotation, and in flat space this will mean that there exist complex coordinates such that the rotation is SU(N).

Another way to think about this result is as follows. In curved space, a single D-brane will preserve half of the supersymmetries if it is a supersymmetric subspace [9,10], a condition which can be stated using the geometric data of complex structure, volume form and so on. (See [11] for a conformal field theory generalization of this statement.) The flat space limit of this condition is just eq. (2.1). In flat space, a single D-brane always preserves half of the supersymmetry, so any hyperplane can be a supersymmetric subspace. Once we use it as such, we constrain the possible complex structures available for other supersymmetric subspaces.

The condition bears a close resemblance to the condition that a curved manifold preserve a supersymmetry – that its holonomy live in a subgroup of SO(d) such as SU(d/2). If we could think of the matrix Γ_D of (2.1) as a discrete holonomy acting on the supersymmetry parameter ε , this analogy would become precise. We will discuss a precise form of this in the last section.

We proceed to discuss some explicit solutions.

2.1. Holomorphic Curves

A 2-brane is supersymmetrically embedded into a holomorphic curve, which in flat space must be a plane v_iW^i (i = 1,...,N). Given K such 2-branes, an SU(N) rotation takes any holomorphic plane into any other. All of these planes preserve a common supersymmetry. Explicitly, embed the first brane into the complex W^1 plane. It now has

$$\Gamma_{D1} = i(a_1^{\dagger} + a^1)(a_1^{\dagger} - a^1)$$

$$= i(1 - 2a_1^{\dagger}a^1).$$
(2.6)

An SU(N) rotation can take this into a general complex plane, with

$$\Gamma_{D,R} = i(1 - 2v^* \cdot a^{\dagger}v \cdot a). \tag{2.7}$$

All of these preserve the state $|0\rangle$, as well as the state $\prod_{k=1}^{N} a_k^{\dagger} |0\rangle$ (since $||v||^2 = 1$).*

Without loss of generality, let us consider N=2. The supersymmetries left by the first brane is one from the $\mathbf{2}_L$ and one from the $\mathbf{2}_R$ representations of $SU(2)_L \times SU(2)_R \sim SO(4)$. The SU(2) above is one of these SU(2)'s. There is in fact another configuration that does not break all the supersymmetry which is a diagonal rotation in both SU(2)'s; this corresponds to two separate rotations of the plane parallel and perpendicular to the first brane, but there is a set of complex coordinates in which this is an SU(2) rotation.

2.2. n-branes in dimension 2n

In 2n dimensions, it is natural to embed n dimensions of the world-volume of the first brane into Re Z^i . Again, other branes related to the first by SU(n) rotation will preserve a common supersymmetry.

Proof: The first D-brane then has

$$\Gamma_{D1} = \prod_{k=1}^{n} \left(a_k^{\dagger} + a^k \right). \tag{2.8}$$

The condition (2.2) then becomes

$$\prod_{k} \left(a_{k}^{\dagger} + a^{k} \right) \varepsilon = \prod_{k} \left(R^{\dagger}_{k}^{l} a_{l}^{\dagger} + R^{k}_{l} a^{l} \right) \varepsilon. \tag{2.9}$$

Now $\varepsilon = |0\rangle$ is a solution: if R is special unitary, both sides of (2.9) are equal to $\prod_k a_k^{\dagger} |0\rangle$.

The same argument shows that $\prod_k a_k^{\dagger} |0\rangle$ is another unbroken supersymmetry. For n even these spinors have the same chirality and two chiral supersymmetries are preserved, while for n odd they have opposite chirality and one supersymmetry is preserved.

^{*} We thank A. Morosov for independently verifying that such a configuration will preserve a supersymmetry.

2.3. Remarks

The simplest example of the above solutions consists of two 2-branes in four dimensions $X^{6,7,8,9}$. Begin with two 2-branes oriented along the $X^{6,8}$ -axes, and then rotate one of them by the angle θ in the X^6X^7 plane and $-\theta$ in the X^8X^9 plane. We will often use this case in the discussion which follows.

In terms of section 2.2, we define the complex coordinates $Z^1=X^6+iX^7$ and $Z^2=X^8+iX^9$. The branes are related by $Z^1\to e^{i\theta}Z^1$ and $Z^2\to e^{-i\theta}Z^2$.

Conversely, in the construction of section 2.1, the complex coordinates are $W^1 = X^6 + iX^8$ and $W^2 = X^7 - iX^9$ (the orientation must be consistent, dictating the minus sign). The rotation is then a real element of $SO(2) \subset SU(2)$.

It is clear that 1-branes can not be oriented at non-trivial angles in a supersymmetric fashion. There are also many supersymmetric configurations consisting of branes of differing dimensions. Many of these are T-dual to the above examples.

Type I theory also contains 9-branes which require the condition $\tilde{\varepsilon} = \varepsilon$. This is compatible with the 2-brane solutions but not the N-brane solutions in general.

We can make the analogous conjecture for the exceptional reduced holonomy groups G_2 or Spin(7) as well – configurations of branes related by rotations in these groups will preserve 1/8 or 1/16 supersymmetry.

3. World-sheet results

Given two branes related by a rotation R, we work with complex coordinates z^i in which R is diagonal with eigenvalues $\exp i\theta_i$, and define corresponding world-sheet fields Z^i . Then the boundary conditions on stretched open strings are (in the notation of section 2.2)

Re
$$\frac{\partial}{\partial \sigma} Z^{i}|_{\sigma=0} = 0$$

Im $Z^{i}|_{\sigma=0} = 0$
Re $e^{i\theta_{i}} \frac{\partial}{\partial \sigma} Z^{i}|_{\sigma=\pi} = 0$
Im $e^{i\theta_{i}} Z^{i}|_{\sigma=\pi} = 0$. (3.1)

These shift both bosonic and fermionic mode expansions by θ_i/π .

3.1. n=2

Let us study the example of section 2.3, and let $\alpha \equiv \theta_1/\pi > 0$. When $\alpha = 1/2$, the NS sector fermions become periodic, and the space-time massless bosons are in the spinor of SO(4), reduced to a doublet by the GSO projection. We can follow the spectrum from this point by continuously varying α .

The mode expansion for the (complex) bosonic field is

$$Z(w, \bar{w}) = \sum_{m \in \mathbb{Z}} \left\{ x_{-\alpha+m} e^{i(m-\alpha)w} + \tilde{x}_{\alpha+m} e^{-i(m+\alpha)\bar{w}} \right\}$$
(3.2)

where $w = \sigma + \tau$, $\bar{w} = \sigma - \tau$ and $x, \tilde{x} \in \mathbb{R}$. The oscillators have non-zero commutators

$$[x_{-r}, \tilde{x}_s] = \frac{1}{r} \delta_{r,s}.$$

In the NS sector, the fermion modes are shifted by a further 1/2. The vacuum energy is found to be $\Delta E = -\frac{1}{2} + \alpha$.

We can form states by raising the vacuum with the oscillators $x_{-\alpha-m}$, $\tilde{x}_{\alpha-1-m}$, $\psi_{-\alpha-1/2-m}$, and $\tilde{\psi}_{\alpha-1/2-m}$ for $m \geq 0$, where we are assuming $0 \geq \theta \geq \pi/2$. The GSO-like projection will lift the vacuum as well as bosonic excitations of it. Thus we get massless bosonic states:

$$\tilde{\psi}_{\alpha-1/2}^{i}|0\rangle \quad (i=1,2) \tag{3.3}$$

which we expect to be localized at the crossing of the two branes. This result may be found in an operator formalism as well; the GSO projection is found by imposing mutual locality. For example, in the NS sector, there are the vertex operators

$$\sigma_{+}^{1}\sigma_{-}^{2}e^{i(1-\alpha)H_{1}}e^{i\alpha H_{2}}e^{-\phi}
\sigma_{+}^{1}\sigma_{-}^{2}e^{-i\alpha H_{1}}e^{-i(1-\alpha)H_{2}}e^{-\phi}$$
(3.4)

in this sector. The operator-state mapping should be clear from eqs. (3.3) and (3.4). The sector in which the open string between the 2 D-branes is oriented in the other direction will have the same amount of bosonic and fermionic spacetime fields. These will have the bosonic twist operators $\sigma_{-}^{1}\sigma_{+}^{2}$ in their vertex operators.

For small α , there are towers of bosonic states

$$\tilde{\psi}_{\alpha-1/2}^{i} \left(x_{-\alpha}^{1}\right)^{n_{1}} \left(x_{-\alpha}^{2}\right)^{n_{2}} |0\rangle; \quad n_{1} + n_{2} = N$$

$$\psi_{-\alpha-1/2}^{i} \left(x_{-\alpha}^{1}\right)^{n_{1}} \left(x_{-\alpha}^{2}\right)^{n_{2}} |0\rangle; \quad n_{1} + n_{2} = N - 2$$

$$\psi_{-1/2}^{\mu} \left(x_{-\alpha}^{1}\right)^{n_{1}} \left(x_{-\alpha}^{2}\right)^{n_{2}} |0\rangle; \quad n_{1} + n_{2} = N - 1$$
(3.5)

each with energy $E_N = \alpha N$. The corresponding towers in the Ramond sector are

$$(x_{-\alpha}^{1})^{n_{1}} (x_{-\alpha}^{2})^{n_{2}} | \pm \mp \rangle; \quad n_{1} + n_{2} = N$$

$$\psi_{-\alpha}^{1} \psi_{-\alpha}^{2} (x_{-\alpha}^{1})^{n_{1}} (x_{-\alpha}^{2})^{n_{2}} | \pm \mp \rangle; \quad n_{1} + n_{2} = N - 2$$

$$\psi_{-\alpha}^{i} (x_{-\alpha}^{1})^{n_{1}} (x_{-\alpha}^{2})^{n_{2}} | \pm \pm \rangle; \quad n_{1} + n_{2} = N - 1$$

$$(3.6)$$

At the massless level, we find a hypermultiplet in six dimensions.³ This spectrum appears to be anomalous; the resolution of this problem is discussed in [12,13]. At the Nth mass level, there are 2N massive hypermultiplets (\equiv hyper+vector).

As $\theta \to 0$, this tower of states comes down, and fills out eight-dimensional representations. The DN strings are confined with a potential $(\theta X)^2/\alpha'$ which is becoming more shallow. We will return to this in section 4.

3.2.
$$n = 3$$

Next, we consider the configuration of section 2.2 with n=3. This is particularly interesting, because the two branes can intersect in 3+1 dimensions. Let us discuss a pair of 6-branes in type IIA intersecting on a 4-plane; we will find that the open strings joining them produce chiral 4D matter content on the intersection.

This may seem counterintuitive. One orientation of the open string has $U(1) \times U(1)$ charge (1, -1), while the other has charge (-1, 1). Which charge should we assign to the chiral field? Equivalently, what correlates the world-sheet orientation with the space-time chirality of the fermions? The GSO projection, of course, but how does it see world-sheet orientation?

Given an oriented 4k+2 dimensional space, we can define a 'left-handed rotation' as follows. Reduce the rotation to normal form, a product of 2k+1 rotations in two-dimensional planes. To each two-plane, associate the two-form $dx \wedge dy$ where the sign

³ Here we count both orientations of the open strings.

is chosen to make the rotation in the (x, y) plane left handed. A left-handed rotation is then one in which the wedge product of these forms is positive. The GSO projection will correlate the four-dimensional chirality with the chirality of the rotation relating the two branes and thus the orientation.

More explicitly, take a rotation R which in diagonal form is $R = diag\left(e^{i\theta_1}, e^{i\theta_2}, e^{i\theta_3}\right)$, where $\sum_i \theta_i = 0$. The boundary conditions are as written in eqs. (3.1). The mode expansions for the three Z^i are as in (3.2), each depending on the appropriate α_i . The ground state energy is $\Delta E = -1/2 + \alpha_1 + \alpha_2$, assuming $\alpha_{1,2} \geq 0$. In the NS sector, we find a single massless spacetime boson $\tilde{\psi}^3_{-1/2-\alpha_3}|0\rangle$ and a single spacetime massless fermion. There is a tower of states at energies $E = n_1\alpha_1 + n_2\alpha_2$, which can be interpreted as arising from a harmonic oscillator potential $V = 1/2\sum_i \left(\frac{\theta_i Z^i}{\pi}\right)^2$.

The vertex operators for the open string that stretches between the two 6-branes are: Orientation 1:

$$V_b = \sigma_+ e^{i(\alpha_1 H_2 + \alpha_2 H_3 + (1 - \alpha_1 - \alpha_2)H_4)},$$

$$V_f = \sigma_- e^{i((-\frac{1}{2} + \alpha_1)H_2 + (-\frac{1}{2} + \alpha_2)H_3 + (\frac{1}{2} - \alpha_1 - \alpha_2)H_4)} e^{\pm i(H_0 + H_1)}$$
(3.7)

Orientation 2:

$$V_b = \sigma_- e^{-i(\alpha_1 H_2 + \alpha_2 H_3 + (1 - \alpha_1 - \alpha_2) H_4)},$$

$$V_f = \sigma_- e^{i((\frac{1}{2} - \alpha_1) H_2 + (\frac{1}{2} - \alpha_2) H_3 + (-\frac{1}{2} + \alpha_1 + \alpha_2) H_4)} e^{\pm i(H_0 - H_1)}$$
(3.8)

where for future reference we have not yet gone to lightcone and thus have two fermionic states in each sector.

We have obtained one physical fermion and one physical boson degrees of freedom in each orientation of the string that stretches between the two 6-branes. These assemble themselves into an N=1 chiral multiplet. The GSO projection can be verified by requiring locality with respect to the supercharge. This makes the space time fermion twist operator $e^{\pm \frac{i}{2}(H_0+H_1)}$ in one orientation and $e^{\pm \frac{i}{2}(H_0-H_1)}$ in the other (we have not yet gone to the lightcone to cut the number of states by half). The charge assigned to the chiral field is that of the orientation that has the left handed fermion.

The theory is anomalous, but as discussed in [12], this is compensated by 6-brane bulk contributions. To show this, we just need to reproduce eq. (2.3) of Ref. [12], and the rest is

as explained there. We need to show that $I = ch_{(1,-1)}(F)\hat{A}(R) = 2ch_1(F_1)ch_{-1}(F_2)\hat{A}(R)$ where on the LHS we have the Chern classes of both U(1) fields in the (1,-1) representation and on the RHS the product of each the Chern classes in each U(1) in the one dimensional representation with the appropriate charge. To show this we expand the relevant part of I, which is the 6-form part, and just show the equality order by order.

More generally, if we include N_1 (N_2) branes of each type, we find $U(N_1) \times U(N_2)$ gauge theory. The chiral multiplet now transforms in the ($\mathbf{N_1}, \mathbf{\bar{N}_2}$) representation.

4. Space-time Interpretation

The parallel separation of two Dp-branes can also be described as the Higgs mechanism in the p+1-dimensional field theory that describes the low energy excitations of the two branes when they are coincident. In this theory, we can describe two branes intersecting at an angle by turning on a linearly rising vev. For example, if we let X^6 be a coordinate in the branes and X^7 perpendicular, then we should turn on the corresponding fields $X^7 = X^6\sigma_3 \tan \theta$.

When the angle θ is small, this is clearly well-described by the low-energy effective field theory. We now show that such a vev will truncate the spectrum as we expect and will localize the remaining excitations around the intersection of the two D-branes.

Even though we are rotating one of the branes in such a way that the system is still BPS saturated, there is no corresponding modulus in the p+1-dimensional field theory. This is because there is no boundary condition on the field theory at infinity that parameterizes different angles between the branes, and therefore there is no moduli space and no massless particle associated with it. One could compactify these directions on, say, a torus with the branes wrapping on cycles; however, the angle between the branes is related to the shape of the torus, and is not an independent parameter of the brane system.

Let us consider the example of section 2.1, adding four additional dimensions to the branes to make them 5-branes each with a 2-plane in \mathbb{R}^4 . We will be interested in the field $W^2 = X^7 - iX^9$ which gives the separation of the two D5-branes; this is a hermitian matrix which we take to be $W^2 = \phi \Sigma$ with $\Sigma = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Rotating one brane relative to

the other will be given (to lowest order in θ) by $\phi = \theta \begin{pmatrix} Z_1 & \\ & -Z_1 \end{pmatrix}$.

To check that we don't have to turn on more fields, we study the unbroken supersymmetries of this configuration. It is sufficient to consider the D=10 supersymmetry transformation, reduced to 6-dimensions. The transformation law for the gaugino is

$$\delta \chi^a = F_{mn} \Gamma^{mn} \eta.$$

Now, the ϕ given above lifts to 10-dimensions as the gauge field A^{z_2} ; hence the non-zero components of the field strength are $F_{z_1\bar{z}_2} = \partial_{z_1}A^{z_2}$ and $F_{\bar{z}_1z_2}$. Thus, the remaining supersymmetries η are such that they are annihilated by $a_1^{\dagger}a_2$ and $a_2^{\dagger}a_1$. This gives the same susy |0> that remains from 2 D-branes at an angle as was discussed above. So it is consistent with supersymmetry to turn on only this field.

To show which modes become massless, and to display the localization of states, we insert this background into the low energy 6-dimensional worldvolume Yang-Mills theory, and expand around it. The important feature of the theory for this purpose is the d=6 potential $\operatorname{tr}[X^{\mu}, X^{\nu}]^2$ as well as the gauge couplings to the scalars $\operatorname{tr}[A_{\mu}, X^{\nu}]$. After rotation, we will derive an effective 4D theory, in which the scalars and two of the 6D gauge fields become scalars in 4D. As in standard Kaluza-Klein, if we take the linear fluctuation to be independent of the 4D space-time, we obtain two-dimensional differential equations whose eigenvalues are the masses of the 4D particles. These 2D solutions are trapped inside a harmonic oscillator potential well and are thus localized to the intersection of the branes.

Let us briefly discuss a simple example, the gauge field equation of motion. The modes corresponding to open strings between the two branes are $\delta\phi=\begin{pmatrix}0&f_1\\f_2&0\end{pmatrix}$ and $\delta A_\mu=\begin{pmatrix}0&A_\mu^1\\A_\mu^2&0\end{pmatrix}$. One finds, for example,

$$\partial^2 \delta A^i_\mu + \left(\frac{\theta}{\pi \alpha'}\right)^2 |z|^2 \delta A^i_\mu = 0. \tag{4.1}$$

The background produces a space dependent mass $\theta|z|/\pi\alpha'$. Clearly normal modes of (4.1) will be localized around z=0; they have the spacing for the tower of states found in section 3.

We used the $\alpha' \to 0$ limit on the 5-brane. How much can we trust it? In general, since there are linearly rising vev's, we expect that higher derivative terms will be important;

some of these would be taken into account by considering the full Born-Infeld form of the action. Dangerous higher derivative terms are those that, after taking one derivative of the vev's, are proportional to the rotation angle and are constant throughout the D=6 spacetime, and therefore not suppressed by momentum power counting. Such terms are independent of 6-dimensional space-time and so may shift the zero point energy of the oscillator described above, but will not affect the localization of the states. One might also expect that a low energy effective action description can be trusted as long as the states are not localized to within $\sqrt{\alpha'}$ of the intersection; hence, we assume $\theta << 1$.

5. T-Duality

We will now comment on T-duality. We consider our canonical example with two 2-branes, using the notation of Section 2.2. The first plane lies along Re Z^1 , Re Z^2 , while the second is rotated by $Z^1 \to e^{i\theta}Z^1$, $Z^2 \to e^{-i\theta}Z^2$.

Open strings then have boundary conditions

$$2_{1}: \quad \partial_{n} X^{6,8} = 0$$

$$\partial_{\tau} X^{7,9} = 0$$

$$2_{2}: \quad \partial_{n} \left(\cos \theta \ X^{6,8} \mp \sin \theta \ X^{7,9} \right) = 0$$

$$\partial_{\tau} \left(\pm \sin \theta \ X^{6,8} + \cos \theta \ X^{7,9} \right) = 0.$$
(5.1)

We consider toroidally compactifying $X^{6,7,8,9}$ on T^4 , with nontrivial metric components. In particular, if we want the branes to be consistent with the toroidal compactification, we should take the two T^2 's in T^4 to have moduli $\operatorname{Arg} \tau = \theta$.

If we T-dual in the $X^{6,8}$ directions, we will get a square T^4 (Kähler moduli and complex structure being interchanged), and the two 2-branes will became a 0-brane and a 4-brane. The boundary conditions become

0:
$$\partial_{\tau} X^{6,8} = 0$$

 $\partial_{\tau} X^{7,9} = 0$
4: $\partial_{n} X^{7,9} \mp \cot \theta \ \partial_{\tau} X^{6,8} = 0$
 $\partial_{n} X^{6,8} \pm \cot \theta \ \partial_{\tau} X^{7,9} = 0.$ (5.2)

⁴ For simplicity, we consider here a brane wrapped once around one cycle of the T^2 .

As in Ref. [14], these boundary conditions can be thought of as a 4-brane in a background⁵

$$\hat{F}_{76} = \hat{F}_{98} = -\cot\theta. \tag{5.3}$$

Thus the supersymmetry condition that we found for the 2-branes, namely that the rotation angles for the two planes be equal, is T-dual to an anti-self-dual gauge field background. To see that this makes sense, consider the unbroken supersymmetries for the T-dual system. Here we have:

$$\tilde{\varepsilon} = \varepsilon$$

$$\tilde{\varepsilon} = -\Gamma^6 \Gamma^7 \Gamma^8 \Gamma^9 \varepsilon.$$

$$(5.4)$$

Defining the lowering operators $a_1 = (\Gamma^6 + i\Gamma^7)/2$ and $a_2 = (\Gamma^8 + i\Gamma^9)/2$, the unbroken supersymmetries are $\varepsilon = |0\rangle, a_1^{\dagger} a_2^{\dagger} |0\rangle$. These two Fock states also satisfy $\Gamma^{67} \varepsilon = +\Gamma^{89} \varepsilon$. Now in the presence of a background gauge field, the supersymmetry transformation of the gaugino is

$$\delta \chi = \Gamma^{MN} \hat{F}_{MN} \, \eta \tag{5.5}$$

Given the above remarks, this vanishes iff \hat{F} is anti-self-dual.

In other situations (n > 2), the analogues of eqs. (5.4) and (5.5) contain additional dependence on the background fields. This will be explored in Ref. [15].

6. Higher dimensional interpretation

The two supersymmetries of the Type IIA string are unified into a single elevendimensional spinor η in M-theory[16,17]. Does the condition (2.1) follow from something simple?

We now show that all of the D-brane supersymmetry conditions (2.1), as well as the condition for unbroken supersymmetry in the presence of a NS 5-brane, are unified into the condition

$$\eta = \prod_{i} e_{i}^{\mu} \Gamma_{\mu} \eta \tag{6.1}$$

⁵ We define $\hat{F} = F + B$, where F is the worldvolume gauge field.

where the product over e_i^{μ} includes e^{11} for a brane with winding number around X^{11} (such as the 4-brane) or momentum P^{11} (such as the 0-brane), but does not include e^{11} for others (such as the 2-brane).

Proof: The condition (2.1) is

$$(1 - \Gamma^{11})\eta = \Gamma_D (1 + \Gamma^{11})\eta \tag{6.2}$$

where Γ_D only includes dimensions within the ten. Multiply both sides by Γ_D and use

$$\Gamma_D^2 = \omega \equiv \begin{cases} +1 & \text{for } k = 4l + 2\\ -1 & \text{for } k = 4l \end{cases}$$

$$(6.3)$$

to derive

$$\omega(1+\Gamma^{11})\eta = \Gamma_D(1-\Gamma^{11})\eta. \tag{6.4}$$

Adding the two equations produces (6.1), with Γ^{11} appearing in the product if k=4l.

So, it clearly works for D0, D2, D4 (wound 5-brane) and the 9-brane of Type I. The d = 11 interpretation of D8 is still unclear [18].

The NS IIA 5-brane has chiral world brane susy. This means that the two spinor projections are $\epsilon = \Gamma_D \epsilon$, $\tilde{\epsilon} = \Gamma_D \tilde{\epsilon}$. Since $\eta = \epsilon + \tilde{\epsilon}$ then $\eta = \Gamma_D \eta$ and indeed it is unwound.

For the purposes of this criterion, the D6 brane is unwound. As it is identified with the Kaluza-Klein monopole [19], this may be a bit counterintuitive, but leads to no contradictions.

It would be quite interesting to propose a type IIB version of this.

6.1. D-branes and discrete holonomy

After compactification on a manifold \mathcal{M} , the surviving supersymmetries are those which are preserved by the holonomy group of \mathcal{M} . Can we generalize this statement to compactifications with branes?

A local test for unbroken susy is to scatter a low energy gravitino off of the brane – if it is unaffected, the associated susy is unbroken, but if there is a phase shift, it is broken. Since the phase shift is given by the action of the matrix Γ_D , we want to regard this matrix as discrete holonomy. As we saw above, in d = 11 it is a linear operator on the spinor.

However, this will not be the holonomy of the metric-compatible connection, because the condition for unbroken supersymmetry also depends on the antisymmetric tensor background. We know that we can have supersymmetric backgrounds with no covariantly constant spinor, but with zero modes of the full gravitino transformation law.

The holonomy condition can be adapted to this situation by using the gravitino transformation law to define a modified connection, as was proposed long ago for eleven-dimensional supergravity [20]. The idea is that since the gravitino transformation law

$$\delta\psi_M = \nabla_M \varepsilon - \frac{1}{288} \left(\Gamma_M \, ^{PQRS} - 8\delta_M^P \Gamma^{QRS} \right) F_{PQRS} \, \varepsilon. \tag{6.5}$$

is a linear operator on ε , it can be regarded as a covariant derivative written in terms of a modified connection Ω_M . In terms of the spin connection ω_M and 4-form field strength F,

$$\Omega_M \equiv \frac{1}{4} \omega_M^{mn} \Gamma_{mn} - \frac{1}{288} \left(\Gamma_M^{PQRS} - 8\delta_M^P \Gamma^{QRS} \right) F_{PQRS}. \tag{6.6}$$

It defines the following parallel transport of a spinor:

$$0 = \dot{X}^{M}(t) \left(\nabla_{M} - \frac{1}{288} \left(\Gamma_{M} \, ^{PQRS} - 8\delta_{M}^{P} \Gamma^{QRS} \right) F_{PQRS}(X(t)) \right) \varepsilon(X(t)). \tag{6.7}$$

Integrating this along a path $X^M(t)$ defines the holonomy of the modified connection, or 'generalized holonomy.'

We do not really have to go to d = 11 for this – we could regard the two gravitino transformations in d = 10 as defining a connection which acts on the direct sum of the two spinor parameters. This also allows us to define generalized holonomy for the type IIB theory.

In many cases the field strengths can then be reduced to forms of rank $p \leq 3$, and the modified connection reduces to a connection with torsion, as in [21]. However, this is not true in general.

To apply this to our D-brane case, we should regard the condition (6.1) as defining a discrete generalized holonomy, concentrated at the position of the brane. The statement will then be that in general compactifications with branes, unbroken supersymmetry must commute with the generalized holonomy group.

Does this agree with the generalized holonomy of the solutions of the low energy field theory as defined by (6.7)? It is certainly clear from their BPS nature that both generalized holonomy groups will preserve the same supersymmetries.

We would expect the generalized holonomy group for the soliton solutions to be larger, because the objects have a finite size. Consider for example the supermembrane solution [22]. The d=11 supersymmetry parameter splits into two d=8 spinors, and the gravitino transformation becomes (schematically) $\delta\psi=(D+\Gamma_9F)\varepsilon$. This cancels for one chirality, implying that the spin connection and gauge field terms are equal, and thus the modified connection acting on the other chirality spinor is equal to twice the spin connection. Therefore the generalized holonomy group in this case is SO(8). It would be interesting to see this in the D-brane generalized holonomy group as well, by taking into account corrections in string perturbation theory.

A priori the generalized holonomy group in a specific background could be any subgroup of SO(16, 16), but one might expect constraints on the possibilities. This question is under investigation.

We believe generalized holonomy will be an important element in a deeper understanding of the geometry of supergravity, as in the works [23].

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